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Finite Element Analysis of Lossy Waveguides–Application to Microstrip Lines on Semiconductor Substrate

MICHEL AUBOURG, JEAN-PIERRE VILLOTTE, FRANCK GODON, AND YVES GARAULT

Abstract **—The development of Maxwell's equations is made considering the electromagnetic fields as vector distributions. With the aid of the finite element method, an analysis of lossy shielded inhomogenous waveguides of arbitrary shape is described.**

To solve the complex matrix system an iterative procedure is presented. The method is applied to study the propagation on MIS or Schottky contact microstrip lines.

I. Introduction

THE FINITE ELEMENT method applied to hybrid wave analysis is commonly used to study propagation along quasi-planar lines like microstrip or coplanar lines [1], [2] and along dielectric-loaded waveguides [3] or dielectric waveguides [4]. In these analyses, materials are considered lossless. It is generally sufficient because substrate-like alumina or semi-insulating GaAs suited for microwave integrated circuits are low-loss media. Hence, skin losses are the most important and can be calculated from the current densities on the conductors.

But in monolithic microwave integrated circuits, to decrease the size of elements it is necessary to have a slow wave propagation medium. It is possible to obtain very slow wave propagation by using MIS or Schottky contact realized on a semiconducting substrate [5]–[7]. In this case, the propagation constant and electromagnetic field are complex.

The purpose of this paper is to present, with the aid of the finite element method, a two-dimensional analysis of the propagation in inhomogeneous lossy waveguides. The development of Maxwell's equation is made considering

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the electromagnetic fields as distributions [8]. By applying the finite element method, the analysis leads to a complete matrix system.

Compared to the procedure given in [9], this solution has the advantage of yielding symmetrical matrices in which frequency appears as a parameter.

This numerical method is then applied to study microstrip lines on a semiconducting substrate [10].

II. Derivation of Equations

A. Notation

The wave is assumed to propagate in the positive z direction with

$$\vec{e}(x,y,z,t)=\Re_e\{\tilde{\vec{E}}(x,y,z)e^{j\omega t}\}$$
$$\vec{h}(x,y,z,t)=\Re_e\{\tilde{\vec{H}}(x,y,z)e^{j\omega t}\} \quad (1)$$

where

$$\tilde{\vec{E}}(x,y,z)=\vec{E}(x,y)e^{-\gamma z}$$
$$\tilde{\vec{H}}(x,y,z)=\vec{H}(x,y)e^{-\gamma z} \quad (2a)$$

and

$$\vec{E}(x,y)=\vec{E}_t(x,y)+E_z(x,y)\vec{u}_z$$
$$\vec{H}(x,y)=\vec{H}_t(x,y)+H_z(x,y)\vec{u}_z \quad (2b)$$

and $\gamma=\alpha+j\beta$ is the complex propagation constant.

The waveguide is supposed to be loaded by N lossy dielectrics of permittivity ϵ_i and conductivity σ_i. These values can be functions of the transverse coordinates.

We define

- Ω_i internal domain of permittivity ϵ_i and conductivity σ_i;
- Ω $\bigcup_{i=1}^{N}\Omega_i$, $1\leqslant i\leqslant N$;
- $\Gamma_{i,j}$ interface between media i and j, $1\leqslant i<j\leqslant N$;
- Γ_e electrical walls;
- Γ_m magnetic walls;
- $\vec{n}_{\{{e \atop m}}$ unit vector normal to $\Gamma_{\{{e \atop m}}$ oriented towards the inside of the waveguide;
- $\vec{n}_{\{{i \atop j}}$ unit vector normal to $\Gamma_{i,j}$ oriented towards $\Omega_{\{{i \atop j}}$;

and

$$\epsilon_r(x,y)=\frac{1}{\epsilon_0}\left(\epsilon_i-j\frac{\sigma_i}{\omega}\right),\qquad \text{if } (x,y)\in\Omega_i$$

$$L(x,y)=\begin{cases}1, & \text{if } (x,y)\in\bigcup_{i=1}^{N}\Omega_i\\ 0, & \text{if } (x,y)\notin\bigcup_{i=1}^{N}\Omega_i.\end{cases}$$

B. Transformation of the Equations

The problem is to determine the distributions of the electromagnetic field

$$L(x,y)\quad \tilde{\vec{E}}(x,y,z)\quad\text{and}\quad L(x,y)\quad \tilde{\vec{H}}(x,y,z).$$

So the Maxwell's equations can be written

$$\nabla\times\{L\cdot\tilde{\vec{E}}\}=\tilde{\vec{J}}_{ms}-j\omega\mu_0L\tilde{\vec{H}} \quad (3a)$$
$$\nabla\times\{L\cdot\tilde{\vec{H}}\}=\tilde{\vec{J}}_{es}+j\omega\epsilon_0\epsilon_rL\tilde{\vec{E}}. \quad (3b)$$

$\tilde{\vec{J}}_{ms}$ and $\tilde{\vec{J}}_{es}$ are the current density distributions defined respectively on the magnetic and electrical walls

$$\tilde{\vec{J}}_{ms}=\vec{J}_m\delta_{\Gamma_m}e^{-\gamma z} \quad (4a)$$
$$\tilde{\vec{J}}_{es}=\vec{J}_e\delta_{\Gamma_e}e^{-\gamma z}. \quad (4b)$$

$\delta_{\Gamma_e}(\Gamma_m)$ indicates that the support of $\vec{J}_e(\vec{J}_m)$ is $\Gamma_e(\Gamma_m)$.

After a first development, (3a) leads to

$$\langle\nabla\times L\vec{E},\ \vec{\phi}\rangle+\gamma\langle L\vec{E}\times\vec{u}_z,\ \vec{\phi}\rangle=\langle\vec{J}_m\delta_{\Gamma_m},\ \vec{\phi}\rangle - j\omega\mu_0\langle L\vec{H},\ \vec{\phi}\rangle \quad (5)$$

where $\vec{\phi}$ are the test functions, i.e.,

$$\langle\nabla\times L\vec{E},\ \phi\rangle=\langle L\vec{E},\ \{\nabla\times\}\vec{\phi}\rangle.$$

($\{\nabla\times\}$ means that the operator $\nabla\times$ is taken in the sense of functions.)

The relation (5) is equivalent to

$$\iint_{R^2}L\vec{E}\cdot\{\nabla\times\}\vec{\phi}\,dx\,dy+\gamma\iint_{R^2}(L\vec{E}\times\vec{u}_z)\cdot\vec{\phi}\,dx\,dy = \int_{\Gamma_m}\vec{J}_m\cdot\vec{\phi}\,d\Gamma_m-j\omega\mu_0\iint_{R^2}L\vec{H}\cdot\vec{\phi}\,dx\,dy. \quad (6)$$

Integrating the first term by parts we have

$$\iint_{R^2}L\vec{E}\cdot\{\nabla\times\}\vec{\phi}\,dx\,dy=\iint_{R^2}L(\{\nabla\times\}\vec{E})\cdot\vec{\phi}\,dx\,dy + \int_{\Gamma_e}(\vec{n}_e\times\vec{E})\cdot\vec{\phi}\,d\Gamma_e+\int_{\Gamma_m}(\vec{n}_m\times\vec{E})\cdot\vec{\phi}\,d\Gamma_m + \sum_{i<j}\int_{\Gamma_{i,j}}(\vec{n}_i\times\vec{E}_i+\vec{n}_j\times\vec{E}_j)\cdot\vec{\phi}\,d\Gamma_{i,j} \quad (7)$$

and writing equality between distributions, (6) leads to

$$\{\nabla\times\}\vec{E}+\gamma\vec{E}\times\vec{u}_z=-j\omega\mu_0\vec{H} \quad (8a)$$
$$\vec{n}_e\times\vec{E}=0,\qquad \text{on } \Gamma_e \quad (8b)$$
$$\vec{n}_e\times\vec{E}=\vec{J}_m,\qquad \text{on } \Gamma_m \quad (8c)$$
$$\vec{n}_i\times\vec{E}_i+\vec{n}_j\times\vec{E}_j=0,\qquad \text{on } \Gamma_{i,j},\ 1\leqslant i<j\leqslant N. \quad (8d)$$

These relations show that the first Maxwell's equation is verified (in the sense of functions) (8a), the tangential component of $\vec{E}$ is equal to 0 on an electrical wall (8b), and is continuous at an interface (8d). The relation (8c) is an identity and can be used to define $\vec{J}_m$. Similar derivation can be made from (3b). To study the propagation it is sufficient to solve (3a) and (3b) in which $\vec{J}_m$ and $\vec{J}_s$ are eliminated and the supports of the distributions are reduced to Ω_m and $\Omega_e(\Omega_{\{{e \atop m}}=R^2-\Gamma_{\{{e \atop m}})$ because the currents are induced.

Taking into account transverse and longitudinal distributions, the problem to solve is

$$\nabla \times (L\vec{E}_t) = -j\omega\mu_0 L H_z \vec{u}_z, \qquad \text{on } \Omega_m \tag{9a}$$

$$\nabla \times (L\vec{H}_t) = j\omega\epsilon_0\epsilon_r L E_z \vec{u}_z, \qquad \text{on } \Omega_e \tag{9b}$$

with the relations between components defined by

$$\nabla \times (L E_z \vec{u}_z) + \gamma L\vec{E}_t \times \vec{u}_z = -j\omega\mu_0 L\vec{H}_t \tag{10a}$$

$$\nabla \times (L H_z \vec{u}_z) + \gamma L\vec{H}_t \times \vec{u}_z = j\omega\epsilon_0\epsilon_r L\vec{E}_t. \tag{10b}$$

If the continuities of E_z and H_z are imposed on $\bar{\Omega}$, and if $E_z = 0$ on Γ_e and $H_z = 0$ on Γ_m, the following relations are equivalent to (10)

$$\left.\begin{aligned} k_c^2 \vec{E}_t &= -\gamma\{\nabla\}E_z - j\omega_0\mu_0\{\nabla\}H_z \times \vec{u}_z \quad &(11\text{a}) \\ k_c^2 \vec{H}_t &= -\gamma\{\nabla\}H_z + j\omega\epsilon_0\epsilon_r\{\nabla\}E_z \times \vec{u}_z \quad &(11\text{b}) \end{aligned}\right\}, \quad \text{on } \Omega_i$$

$$k_c^2 = \frac{\omega^2}{c^2}\epsilon_r + \gamma^2 \qquad c^2 = \frac{1}{\epsilon_0\mu_0}.$$

For numerical procedure it is more convenient to use the reduced longitudinal components ψ and Φ defined by

$$\psi = \sqrt{\epsilon_0}\, E_z \quad \text{and} \quad \Phi = \sqrt{\mu_0}\, H_z.$$

If $\mathcal{V}$ is the space vector of continuous functions on $\bar{\Omega}$, $\mathcal{V}_e$ the subspace of $\mathcal{V}$ equal to 0 on Γ_e, $\mathcal{V}_m$ the subspace of $\mathcal{V}$ equal to 0 on Γ_m, and if $\phi_{(m}^{(e}$ are, respectively, the test functions of $\mathcal{V}_e$ and $\mathcal{V}_m$, the relations (9a) and (9b) are equivalent to

$$\iint_\Omega \sqrt{\epsilon_0}\,\vec{E}_t \cdot \{\nabla \times\}(\phi_m \vec{u}_z)\,dx\,dy = -j\frac{\omega}{c}\iint_\Omega \Phi\phi_m\,dx\,dy \tag{12a}$$

$$\iint_\Omega \sqrt{\mu_0}\,\vec{H}_t \cdot \{\nabla \times\}(\phi_e \vec{u}_z)\,dx\,dy = j\frac{\omega}{c}\iint_\Omega \epsilon_r\psi\phi_e\,dx\,dy \tag{12b}$$

and taking into account (11), the final form of the relations is

$$\iint_\Omega \frac{1}{k_c^2}\{\nabla\}\Phi\cdot\{\nabla\}\phi_m\,dx\,dy - j\frac{\gamma c}{\omega}\iint_\Omega \frac{1}{k_c^2}(\{\nabla\}\psi \times \{\nabla\}\phi_m)\cdot\vec{u}_z\,dx\,dy = \iint_\Omega \Phi\phi_m\,dx\,dy \tag{13a}$$

$$\iint_\Omega \frac{1}{k_c^2}\epsilon_r\{\nabla\}\psi\cdot\{\nabla\}\phi_e\,dx\,dy - j\frac{\gamma c}{\omega}\iint_\Omega \frac{1}{k_c^2}(\{\nabla\}\phi_e \times \{\nabla\}\Phi)\cdot\vec{u}_z\,dx\,dy = \iint_\Omega \epsilon_r\psi\phi_e\,dx\,dy. \tag{13b}$$

This formulation has the advantage of being directly deduced from Maxwell's equations.

III. Application of the Finite Element Method

The finite element method is well known [1]–[3].

The method consists of dividing the cross section of the waveguide into triangular subdomains. The unknown solution is approximated by polynomials. Lagrange polynomials are chosen as shape functions. A k degree polynomial $P(x, y)$ has $d = (k+1)(k+2)/2$ degrees of freedom. This polynomial is defined if the values are known at d points.

The polynomials $P_\mu(x, y)$ defined in (14) are associated at each node located by $\mu(\mu_1, \mu_2, \mu_3)$. d nodes are defined for each mesh $\mu_i = k\lambda_i$, $1 \leqslant i \leqslant 3$, and λ_i are the area coordinates

$$P_\mu(x, y) = q_{\mu_1}(\lambda_1)q_{\mu_2}(\lambda_2)q_{\mu_3}(\lambda_3) = 1 \quad \text{if} \quad \mu_i = 0 \tag{14}$$

with

$$q_{\mu_i}(\lambda_i) = \frac{1}{\mu_i!}k\lambda_i(k\lambda_i - 1)\cdots(k\lambda_i - (\mu_i - 1)), \qquad \text{if } \mu_i \geqslant 1.$$

At each node i of the triangular mesh K, the basis function associated is

$$W_i(x, y) = \begin{cases} P_\mu(x, y) & \forall (x, y) \in K \\ 0 & \forall (x, y) \notin K. \end{cases} \tag{15}$$

So ψ and Φ can be developed with the basis functions

$$\psi(x, y) = \sum_{j_e} \psi_{j_e} W_{j_e}(x, y), \qquad 1 \leqslant j_e \leqslant S,$$

$$W_{j_e}(x, y) = 0, \qquad \text{on } \Gamma_e, \tag{16a}$$

$$\Phi(x, y) = \sum_{j_m} \Phi_{j_m} W_{j_m}(x, y), \qquad 1 \leqslant j_m \leqslant S,$$

$$W_{j_m}(x, y) = 0, \qquad \text{on } \Gamma_m \tag{16b}$$

where S is the number of nodes.

Equations (13a) and (13b) then become

$$\sum_{j_e} \psi_{j_e} a_e + \sum_{j_m} \psi_{j_m} l(\Phi_e, W_{j_m}) = \sum_{j_e} \psi_{j_e} b_e \tag{17a}$$

$$\sum_{j_m} \Phi_{j_m} a_m + \sum_{j_e} \psi_{j_e} l(W_{j_e}, \phi_m) = \sum_{j_m} \Phi_{j_m} b_m \tag{17b}$$

with

$$a_e = \iint_\Omega \frac{1}{k_c^2}\epsilon_r\{\nabla\}\phi_e\cdot\{\nabla\}W_{je}\,dx\,dy$$

$$a_m = \iint_\Omega \frac{1}{k_c^2}\{\nabla\}\phi_m\{\nabla\}W_{j_m}\,dx\,dy$$

$$b_e = \iint_\Omega \epsilon_r\phi_e\cdot W_{j_e}\,dx\,dy$$

$$b_m = \iint_\Omega \phi_m\cdot W_{j_m}\,dx\,dy$$

$$l(u, v) = -j\frac{\gamma c}{\omega}\iint_\Omega \frac{1}{k_c^2}(\{\nabla\}u \times \{\nabla\}v)\cdot\vec{u}_z\,dx\,dy.$$

The basis functions W_{j_e} are respectively used as functions ϕ_e and ϕ_m.

If A_e, A_m, B_e, B_m, and L are matrices obtained after development of (17), and if ψ and ϕ represent the column vectors $\{\psi_{j_e}\}$ and $\{\Phi_{j_m}\}$, respectively, (17a) and (17b) can be summarized in the form

$$A_e\psi + C\Phi = B_e\psi$$
$$C^t\psi + A_m\Phi = B_m\Phi \tag{18}$$

with the notation

$$A=\begin{bmatrix} A_e & C \\ C^t & A_m \end{bmatrix} \quad B=\begin{bmatrix} B_e & 0 \\ 0 & B_m \end{bmatrix} \quad \text{and} \quad X=\begin{bmatrix} \psi \\ \Phi \end{bmatrix}. \tag{19}$$

The matrix system to be solved is

$$AX = BX.$$

A and B are complex symmetrical matrices and X is a complex column vector.

In most cases, A and X are functions of ω, (α, β). B is a function of ω only.

If the frequency is chosen as a parameter, it is necessary to calculate α, β, and X.

IV. Numerical Procedure [10]

An iterative procedure is applied on α and β to solve (19).

For a fixed frequency, α_0 and β_0 are chosen. $\lambda(\alpha_0, \beta_0)$ and $Y(\alpha_0, \beta_0)$ are defined by

$$AY = \lambda BY. \tag{20}$$

This relation is equivalent to (19) if λ is equal to unity.

To realize this condition the algorithm of the power method which gives the largest eigenvalue η is applied to the matrix $(A-B)^{-1}B$.

$(A-B)^{-1}BY = \eta Y$ is equivalent to $AY = \left(1+\frac{1}{\eta}\right)BY$.

As η is the largest eigenvalue, $\lambda = 1 + 1/\eta$ extends to one.

The Newton-Ralphson method is applied to obtain the convergence of the iterative procedure.

λ_K is considered as the vector $\begin{pmatrix} R_e(\lambda_K) \\ I_m(\lambda_K) \end{pmatrix}$ and for the $(k+1)$ iterations, α_{k+1} and β_{k+1} are defined by

$$\begin{bmatrix} \alpha_{K+1} \\ \beta_{K+1} \end{bmatrix} = \begin{bmatrix} \alpha_K \\ \beta_K \end{bmatrix} - J_\lambda^{-1}(\alpha_K, \beta_K)\begin{bmatrix} R_e(\lambda_K)-1 \\ I_m(\lambda_K) \end{bmatrix} \tag{21}$$

where J_λ is the Jacobian matrix

$$J_\lambda(\alpha_K, \beta_K) = \begin{bmatrix} \dfrac{\partial R_e(\lambda_K)}{\partial\alpha} & \dfrac{\partial R_e(\lambda_K)}{\partial\beta} \\ \dfrac{\partial I_m(\lambda_K)}{\partial\alpha} & \dfrac{\partial I_m(\lambda_K)}{\partial\beta} \end{bmatrix}. \tag{22}$$

To calculate $\partial\lambda_K/\partial\alpha$ and $\partial\lambda_K/\partial\beta$ it is necessary to write λ in the following form:

$$\lambda(\alpha_k, \beta_K) = \frac{1}{Y^t \cdot B \cdot Y} Y^t \cdot A \cdot Y$$

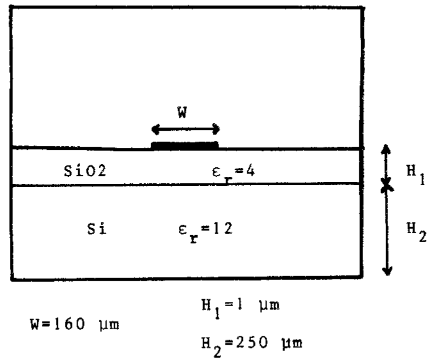

Fig. 1. Cross section of the MIS microstrip line.

so

$$\frac{\partial\lambda}{\partial_{(\beta}^{\alpha}} = \frac{1}{Y^t \cdot B \cdot Y} Y^t \frac{\partial A}{\partial_{(\beta}^{\alpha}} \cdot Y. \tag{23}$$

The advantages of this formulation compared to a precedent analysis [9] are that matrices are symmetrical (which is an important point for numerical procedure) and that frequency appears as a parameter.

V. Applications

This analysis has been applied to MIS microstrip lines. The basic structure is represented in Fig. 1.

Most analyses of microstrip lines on semiconductor substrates [5], [11], [12] have been made with the aid of the parallel-plate waveguide model with a double layer medium [13]. As these are one-dimensional analyses, it is necessary to introduce a correcting factor by analogy with the common strip treatment to take into account the width of the strip [11], [14].

More recently, the spectral domain approach (SDA) [15] has been applied to the MIS structure, and the results are in good agreement with the finite element method.

Theoretical results presented in Fig. 2 are compared with experimental values obtained by Hasegawa *et al.* [5]. Analysis is made around the slow wave region, which is the most interesting region.

For half of the structure, the number of triangular domains was 55. Polynomials of the first degree have been used, so the number of nodes was 34.

Good agreement with experimental results is observed in Fig. 2, even for the attenuation constant, except for the resistivity $\rho = 0.1\ \Omega\cdot$cm. However, theoretical curves show that the attenuation is minimum for this value. In this last case, it is certainly necessary to take into account the metallic losses.

However, a difficulty exists for a first fixed frequency: If the initialization of β_0 is very far from the exact value, the method can converge on a higher order mode. But, after a good result, it is easy to describe a dispersive curve if the initialization is made with the same phase velocity as the one obtained for the previous frequency.

So the first initialization is made by using the value obtained by a finite element quasi-TEM approximation

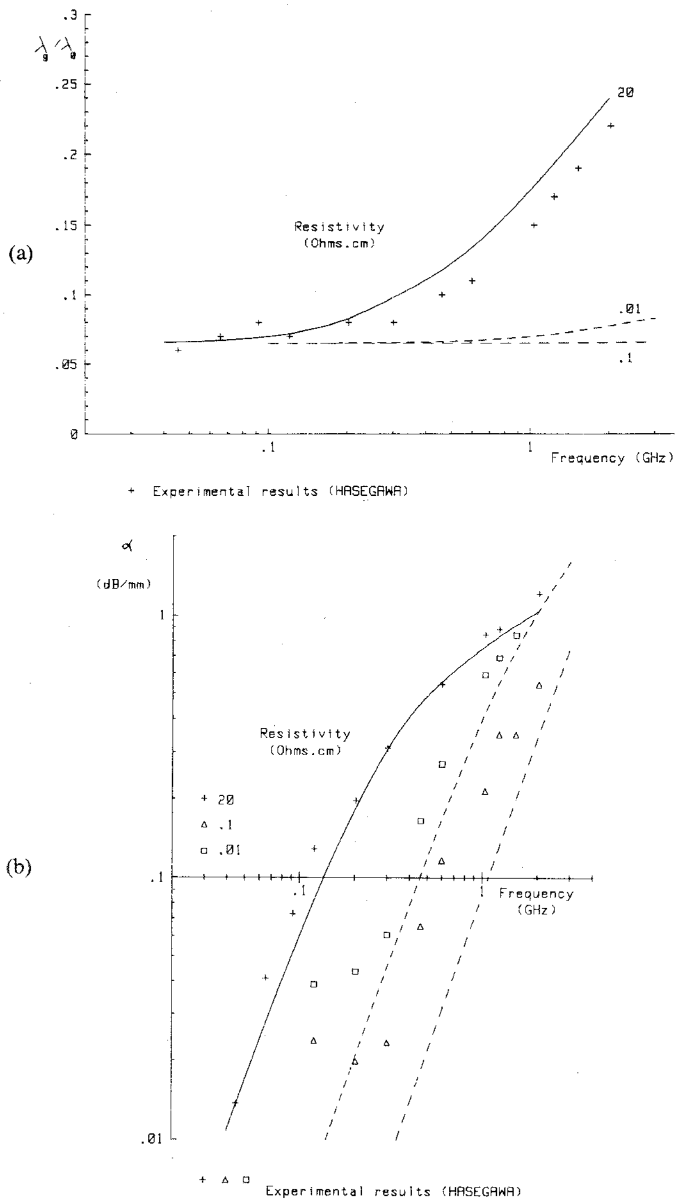

Fig. 2. Comparison between theoretical and experimental results. (a) Slowing factor. (b) Attenuation.

based on the study described in [6]. This analysis can be easily extended to Schottky contact microstrip lines if the depletion layer is represented by a dielectric whose thickness is a function of the applied reverse bias voltage [6], [7], [11], [13], [14].

VI. Conclusion

This numerical method is valid for all analyses of dissipative, shielded waveguides but it has been made, above all, to study the propagation on microstrip or coplanar lines realized on an epitaxial GaAs layer because these are the configurations which present interesting possibilities for monolithic microwave integrated circuits [7].

This method permits the thickness of the strip to be taken into account. This is an important point because the conductor thickness is in the same range as that of the epitaxial layer.

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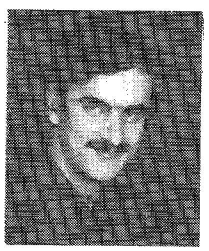

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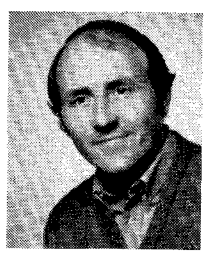

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A Coordinate-Free Approach to Wave Reflection from a Uniaxially Anisotropic Medium

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Abstract—**This paper presents a coordinate-free method of solving the problem of electromagnetic wave reflection from the surface of a uniaxially anisotropic medium. Based on the direct manipulation of vectors, dyadics, and their invariants, the method eliminates the use of coordinate systems. It facilitates solutions and provides results in a greater generality. The paper contains the following results in coordinate-free forms: a) the dispersion equations; b) the directions of field vectors; c) the Poynting vectors (ray vectors) and group velocities; d) the laws of reflection and refraction; and e) the transmission and reflection coefficients. The results are valid for the incident wave having any polarization, and the optic axis of the uniaxial medium being arbitrarily oriented with respect to the interface and the plane of incidence.**

I. Introduction

Because of the rapid advances in technology, wave propagation in anisotropic media such as plasmas, ferrites, etc., has become a subject of intense research [1]–[7]. The emergence of coherent light and optical fibers also makes wave propagation in dielectric crystals a topic of special interest [8]–[12].

In applied electromagnetics, the approach to solutions of various boundary value problems has been the coordinate method [8], [13]–[15]; that is, during the processes of solutions, one or more coordinate systems are introduced. For example, in considering wave propagation in an anisotropic crystal, we formulate and solve the problem with respect to a particular coordinate system—the principal coordinate system of the dielectric tensor [8]. However, when a boundary surface exists, the problem becomes more complex. In this case, two generally inconsistent requirements govern the choice of coordinate system. Inside the crystal, the principal coordinate is preferred, but on the boundary surface, a coordinate system with one of its coordinate planes coinciding with the surface is preferred. Using either system leads to a large number of simultaneous equations and ends in very cumbersome results [16]. Thus, only some special orientations of the optic axis with respect to the interface and the plane of incidence have been considered [17], [18].

In this paper, we shall present a coordinate-free method to solutions of wave reflection from a uniaxially anisotropic medium. We consider only the case when $\bar{\epsilon}$ is a tensor while μ is a scalar. The method applies equally well to the dual case of ferrites. Since the electric and magnetic fields are vector quantities, and they are related by the vector Maxwell equations and constitutive relations, we shall seek vector solutions directly from these vector equations. Based on the direct manipulation of vectors, dyadics, and their invariants, the method eliminates the use of

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